Constraints on the global mass to light ratios and extent of dark matter halos in globular clusters and dwarf spheroidals

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Abstract

The detection of stars in the process of being tidally removed from globular clusters and dwarf spheroidals in the Galaxy's halo provides a strong constraint on their mass to light ratios and on the extent of their possible dark matter halos. If a significant dark matter component existed either within or beyond the observed stellar distribution, then stars would not be removed. We use numerical simulations to study mass loss from two component star clusters orbiting within a deeper potential. We find a global upper limit on the mass to light ratios of globular clusters, M/L$\lesssim$2.5, and rule out the possibility that they have extended halos of low luminosity material. Similarly, the tidal tails of dwarf spheroidals indicates that their dark matter halos must be truncated at ~ 400 pc therefore they have total mass to light ratios $\lesssim$100.

Subject headings. *Dark Matter, Galaxy: Formation. Globular clusters: general. Stars: low mass, brown dwarfs. Galaxies: halos, interactions.*

§1. Introduction

Dark matter plays an important role in the dynamics of a wide range of galactic systems with masses ranging from $10^7 M_\odot$ to $10^{15} M_\odot$. Comprehension of the nature and distribution of dark matter is crucial to understanding the evolution of the Universe. Dwarf spheroidal galaxies are only a few hundred parsecs across and are the smallest systems in which dark matter has been detected (Aaronson 1983). These galaxies have the highest known dark matter densities $\sim 1 M_\odot$ pc^3 and the motions of their stars are completely dominated by dark matter at all radii. Their luminosities are similar to globular clusters, however their sizes are quite different. Globular clusters have core radii $r_c \sim 2$ pc, two orders of magnitude smaller than the dwarf spheroidals. Although the magnitude of the observed stellar velocity dispersions are similar, the dynamical analyses indicate mass to light ratios M/L$\sim 0.5 - 2.5$ in globular clusters (Pryor *et al* 1989) and M/L$\sim 30 - 100$ in dwarf spheroidals (Lake 1990).

The stellar velocities measured for several globular clusters show a steady decline in dispersion with radii, although they never fall to zero. The possibility remains that the luminous component is embedded within a larger sub-luminous dark halo that could extend well beyond the observed "tidal radii" of the cluster (*c.f.* Heggie & Hut 1995, and references within). Extrapolation of the observed mass function of stars in globular clusters have lead to claims that these systems may have large populations of low mass faint stars such as brown dwarfs (Fahlman *et al* 1989, Richer *et al* 1991, Taillet *et al* 1995). Mass segregation would help populate an extended halo of dark matter around clusters and thus the measured M/L ratios are only local values and much higher global values are possible.

Alternative models for dark matter dominated clusters have been proposed by other authors (Peebles 1984, Rosenblatt *et al* 1988). Peebles suggests that two natural scales exist in hierarchical structure formation models in which the mass density is dominated by cold dark matter. In this scenario globular clusters form naturally within the extended dark halos of weakly interacting particles. The baryons can dissipate energy which may result in significant segregation between the light and the mass, leading to global values of M/L as large as 100.

Recent observations by Grillmair *et al* (1995) and Irwin & Hatzidimitriou (1993) have revealed the presence of symmetric tidal tails of stars being gravitationally removed from globular clusters and dwarf spheroidals. In this *letter* we explore the consequences of these observations and how they can be used to constrain a possible component of dark matter within or around these objects. Stellar systems containing dark matter will have larger tidal radii and higher escape velocities, hence the rate of mass loss via evaporation and tidal heating will be lower. We use N-body simulations of star clusters orbiting within a galactic potential to determine the maximum amount of dark matter that could be present in order to be consistent with the observed rate of mass loss.

§2. Tides and Tails

Globular clusters and dwarf spheroidals orbit within the massive dark halo of the Galaxy. These systems are subject to a tidal compression along an axis pointing towards the center of the deeper potential. At a certain distance from the system, a contour of equipotential exists, such that stars will become bound to the Milky Way if they pass beyond this critical surface. Stars will tend to leave the satellites via the extremities of the tidally extended system. Those stars that leave the satellite furthest from the Galaxy will begin to lag behind since their orbital timescale is lower. Likewise, the stars that leave the satellite closest to the Galaxy will form a leading tail due to their higher energies. The stars leave the cluster at close to zero relative velocity, therefore they will follow the same orbital path as the cluster within quite narrow tails and in a direction perpendicular to the tidally extended axis (e.g. Figure 1).

Globular clusters that orbit beyond the galactic disk suffer mass loss primarily via evaporation as stars reach the escape velocity through encounters. The evaporative mass loss from a truncated star cluster can be calculated (e.g. Johnstone 1993 and references within) and is typically between 1–10% per half mass relaxation time:

$$T_{1/2} = \frac{(2/3)^{1/2}(< v^2 >)^{3/2}}{(M/2V_h) < m > 4\pi G^2 \ln\Lambda} \ , \tag{1}$$

where $< v^2 >$ is the mean-square speed of the stars, $< m >$ is the mean mass per star, M is the total mass, V_h is the half mass volume and Λ is the ratio of maximum to minimum encounter distances to be considered (Spitzer 1987).

Over its 15 billion year lifetime, a typical galactic cluster may have lost up to half of its mass via evaporation alone. This stellar debris is detectable using deep imaging observations since the tidal tails are narrow. Grillmair *et al* (1995) use photographic plates to perform star counts beyond the observed tidal radii of a sample of 12 globular clusters. Symmetric tidal tails of escaping stars were detected and as we shall show using N-body simulations, the observed escape rates are roughly consistent with the analytic calculations.

The observed "edges" of the globular clusters are close to the equipotential surface imposed by the Milky Way when mass to light ratios of order unity are adopted. Various dynamical processes will enable stars to move past this surface and become lost to the galaxy. The rate of mass loss depends upon the fraction of stars that have enough energy to pass beyond this tidal radius. For a cluster of fixed physical size, larger mass to light ratios will increase the escape velocity and slow the evaporation rate. As stars diffuse to larger radii they pass through the higher density core less frequently and suffer fewer encounters, thus their ability to reach the escape velocity rapidly decreases.

If the dark matter is distributed on a larger scale than the luminous component, e.g. when the stars are sitting deep within the core of a larger dark matter halo, then the

ability of the visible stars to diffuse to large radii would be reduced. In this case the mass to light ratio within the visible extent of the cluster may not be greatly increased, but the total mass could be many times greater than the mass inferred from the stellar dynamics within the optical radius. Consequently, even modest amounts of dark matter will be very effective at containing the visible stars and halting the production of tidal tails.

One caveat is that highly radial orbits might strip a pre-existing dark matter halo, however, many globular clusters have proper motions determined (Cudworth *et al* 1993). For example, M2 (NGC 7089) has very prominent tidal tails, but the measured space velocity would take the cluster on an orbit between 8 kpc and 15 kpc within an isothermal potential. The tidally imposed radius, R_t, of a system orbiting at a distance R_G within a deeper potential is the point at which their mean densities are equal. For point masses $R_t = R_G(m_{globular}/3M_{Galaxy})^{1/3}$, we find tidal radii for M2 of 81 pc for M/L=1 and 117 pc for M/L=3. Here we have taken the luminosity of M2 to be $L_B = 2.5 \times 10^5 L_\odot$ and the mass of the Milky Way as $10^{11} M_\odot (R_G/10\mathrm{kpc})$. These values should be compared with its King model tidal radius of 60 pc. If the observed stars of M2 were moving within the core radius of a dark halo with an isothermal density profile, then the halo would be tidally limited at $R_t \approx R_G \sigma_{Galaxy}/\sigma_{halo}$. For $\sigma_{halo} = 10\mathrm{km\ s}^{-1}$ and $\sigma_{_{Galaxy}} = 150\mathrm{km\ s}^{-1}$ we find that the globular's dark halo could extend to 500 pc and its global M/L~ 50. In order to strip such a halo from M2 would require a pericentric distance of ~ 1 kpc.

The detection of stars being tidally removed from these systems provides strong evidence that their physical extent is close to that indicated by the stellar distribution. In the next section we shall use N-body simulations to support this discussion and to provide more quantitative numbers on the rate of mass loss from stellar systems that contain dark matter.

§3. Constraining the global distribution of dark matter

We construct equilibrium globular clusters by sampling a King profile with core radius 3 pc and tidal radius 60 pc. Our standard cluster contains no dark matter and has a total mass of $2 \times 10^5 M_\odot$ represented by 10,000 particles. Two additional models were constructed such that the stellar component was embedded within a dark matter halo. The dark matter was distributed using a lowered isothermal sphere with core radius 30 pc and an exponential cutoff at 75 and 100 pc. The mass of the dark particles was set to 1/4 of the star particles. These models had M/L values of 1.5 within the optical radius but the global values were 2 and 3 respectively.

The clusters were placed in orbit within a fixed isothermal potential of circular velocity 220 km s^{-1}, such that the apocentric and pericentric distances were 15 kpc and 8 kpc respectively. We evolved the particle distributions using the TREECODE (Barnes & Hut 1986) with tolerance 0.7 and enough time-steps such that each particle travels 3 steps

across the softening length of 1.5 pc at a velocity equal to the central dispersion ~ 10km s^{-1}.

We ran our standard cluster with no dark matter for 10 Gyrs and we found that the mass loss is roughly constant over this period. For this model, $T_{1/2} \sim 2.5 \times 10^8$ years and for a single mass model with these structure parameters the rate of mass loss is about 1% per half mass relaxation time (Johnstone 1993). The amount of mass unbound from the cluster at the end of this calculation was 30%, in reasonable agreement with the analytic estimate. Note that this example resembles the globular cluster M2, but that our results are easily generalised to any star cluster orbiting within a deeper potential.

Simulating a cluster for a Hubble time, even at our limited resolution requires more than 10^5 timesteps. However, since the rate of mass loss is constant we can compare our simulations with the observations of Grillmair *et al* after only a few Gyrs. Figure 1 shows a snapshot of a model with dark matter after 4 Gyrs. The tidal tails of dark matter are clearly visible since roughly 15% of the dark matter has reached the cluster escape velocity via relaxation of our artificially massive dark matter particles. Low mass stars or WIMPS will produce much smaller tidal tails since the relaxation timescale is proportional to $< m >^{-1/2}$.

The imaging data of Grillmair *et al* reveal of order a hundred stars above the background that are escaping from the cluster. This represents less than 1% of the total number of visible stars that are presently bound to the cluster. This is roughly the expected number of stars in the surveyed area for a typical cluster with no dark matter calculated using the evaporation rates from Johnston (1993) for a multi-component mass model.

In order to compare with the observations we smooth our data on a similar scale as Grillmair *et al* and plot iso-density contours of the stars. Figure 2 shows our results for the 3 models. Clearly, even modest amounts of dark matter can slow down or halt the stellar mass loss. The tidal tails from our standard model are quite similar to the observations, however, as the M/L ratios are increased the tails become much less prominent. Tidal extensions are still visible in our model which has a global M/L=2, although the stellar mass loss has been reduced by about a factor of 5 over the standard model with M/L=1. When we increased the mass to light ratio to 3, then the stellar mass loss is reduced to zero. From these simulations we conclude that the total mass to light ratios of globular clusters cannot be any higher than ~ 2.5, otherwise escaping stars would not have been detected. (Similar results were obtained when we distributed the dark matter in an identical way to the stars.)

3.1 Numerical artifacts

How are these results effected by numerical resolution? We are using 10,000 mass points of effective size 1.5 pc to represent a star cluster that has an order of magnitude more stars of much smaller physical size. Evaporation is a diffusion process that is dominated by

distant encounters rather than close, strong scattering events. Therefore the fact that our stars are physically softened does not reduce the evaporation rate significantly. A more important artifact is a higher than expected mass loss rate since our star particle mass is effectively $20M_\odot$. This can only serve to underestimate our constraint on the global M/L values. Our treatment of the dark matter also enhances the evaporation rate of star particles since the dark particles are also artificially massive. As discussed earlier, in real clusters the dark matter is expected to be much lighter than the visible stars, thus we may be underestimating the effects of mass segregation. We do find that the half mass radius of the stars slowly decreases, although the rate of this effect is significantly smaller than analytic calculations yield (Spitzer 1987). More detailed simulations or analytic calculations could yield stronger constraints on the global M/L values.

§4. Discussion and conclusions

Symmetric tidal tails of stars being gravitationally stripped from globular clusters indicates that their mass distributions do not extend beyond the optical radius. This rules out the possibility of large dark matter halos surrounding globular clusters and hence excludes the possibility that these objects formed as density peaks within a cold dark matter type scenario (Peebles 1983, Rosenblatt 1988). We can also use these results to rule out the presence of large quantities of low mass stars either within or beyond the optical radii (Fahlman *et al* 1989, Richer *et al* 1991, Taillet *et al* 1995). A comparison of stellar isophotes beyond the tidal radii with the data of Grillmair *et al* (1995) provides an upper limit on the total mass to light ratios of globular clusters of M/L $\lesssim$3.

Tidal tails have also been observed to extend from the Ursa-Minor dwarf spheroidal (Irwin & Hatzidimitriou 1993). Data from these authors shows a large number of stars beyond the classical tidal radius of 400 pc. Our results can be readily applied to the dwarf spheroidal galaxies and suggests that the dark matter halo of Ursa-Minor must be truncated at the optical radius $\sim$ 400 pc and hence the mass distribution traces the light distribution.

If the dwarf spheroidal galaxies formed within cold dark matter halos then the fact that they are truncated at $\sim$ 400 pc has interesting consequences for cosmology. Namely, for the first time we have obtained an *upper* limit on the mass to light ratio of a dark matter dominated system. The radial velocity dispersion of stars within dwarf spheroidals appears to be fairly constant with radius If we parameterise the halo by an isothermal sphere with 1-d velocity dispersion 10 km s^{-1}, then the virial radius should nominally extend to 15 kpc for a closed Universe. Tidal truncation at the satellites present galacto-centric distance would limit the radial extent of its halo to $\sim$ 5 kpc. In order to strip the halo to within 400 pc would require a pericentric distance of $\sim$ 6 kpc. Proper motion studies of this galaxy would be extremely interesting and could be used to distinguish between halo formation

within a low density Universe, or tidal stripping on an elongated orbit.

Acknowledgments

I would like to acknowledge useful discussions with Fabio Governato and Doug Johnstone. The numerical simulations were performed and analysed using the resources of the HPCC group at Seattle, funded by NASA through the LTSA and HPCC/ESS programs.

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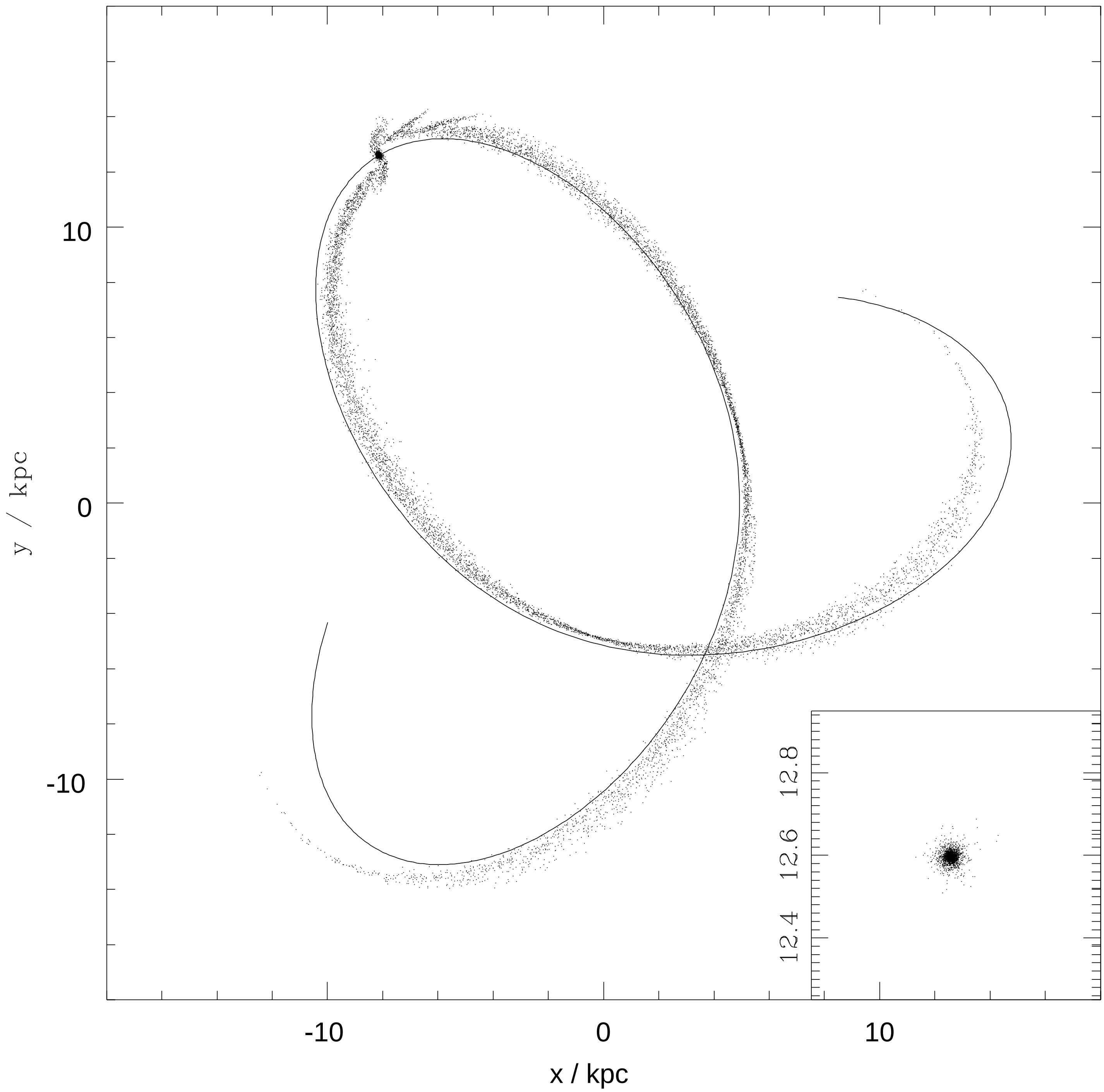
Figure captions

Figure 1. The particle distribution from one of our simulations after 4 Gyrs of evolution. The viewpoint is looking down upon the orbital plane and the curve depicts the cluster's orbit over 0.5 Gyrs. In this simulation the model globular cluster has a dark halo that extends beyond the stellar component. The presence of the dark halo has reduced the stellar mass loss to practically zero. The tidal tails contain 10% of the dark matter particles

which are lost primarily due to relaxation because of their artifically large masses. The inset plot shows the distribution of stars which shows no evidence of mass loss into tidal tails.

Figure 2. Contour plots of the stellar distribution in the immediate vicinity of the globular cluster which should be compared with Grillmair *et al* , in particular their Figure 13. The axis scale is in parsecs and the stellar density has been smoothed in projection using a Gaussian of width 5 pc. The arrows indicate the direction to the center of the galactic potential. (a) The standard model cluster without dark matter. (b) and (c) are for the model cluster with a dark matter halo such that the total mass to light ratios are 2 and 3 respectively. The similarity between the observations and the mass loss rate from our standard cluster in (a) provides conformation that our understanding of stellar dynamics and the evaporation process in star clusters is close to being correct.



10
0
-10
y / kpc
-10
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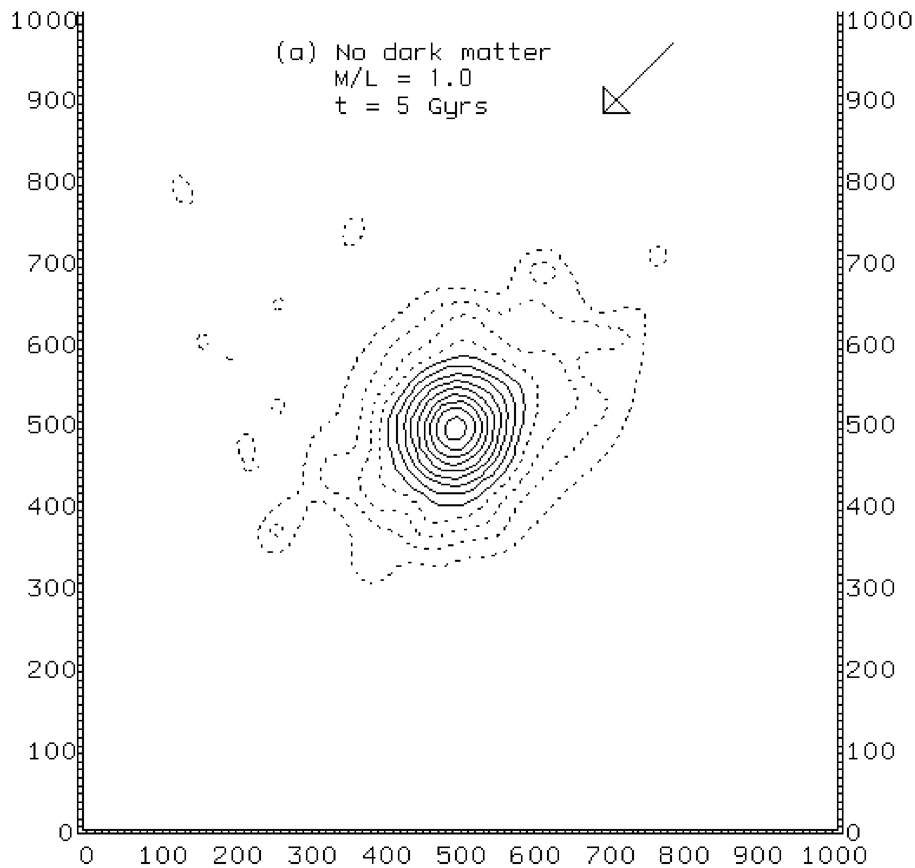
12.8
12.6
12.4


(a) No dark matter
M/L = 1.0
t = 5 Gyrs
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100
200
300
400
500
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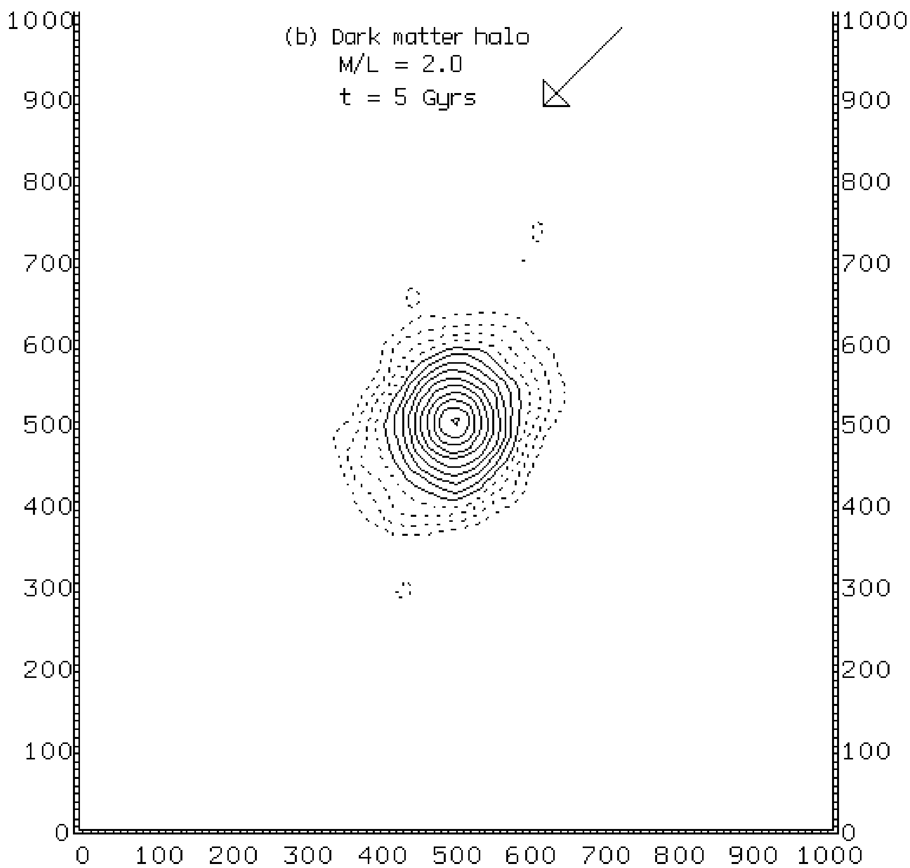
700
800
900
1000


(b) Dark matter halo
M/L = 2.0
t = 5 Gyrs
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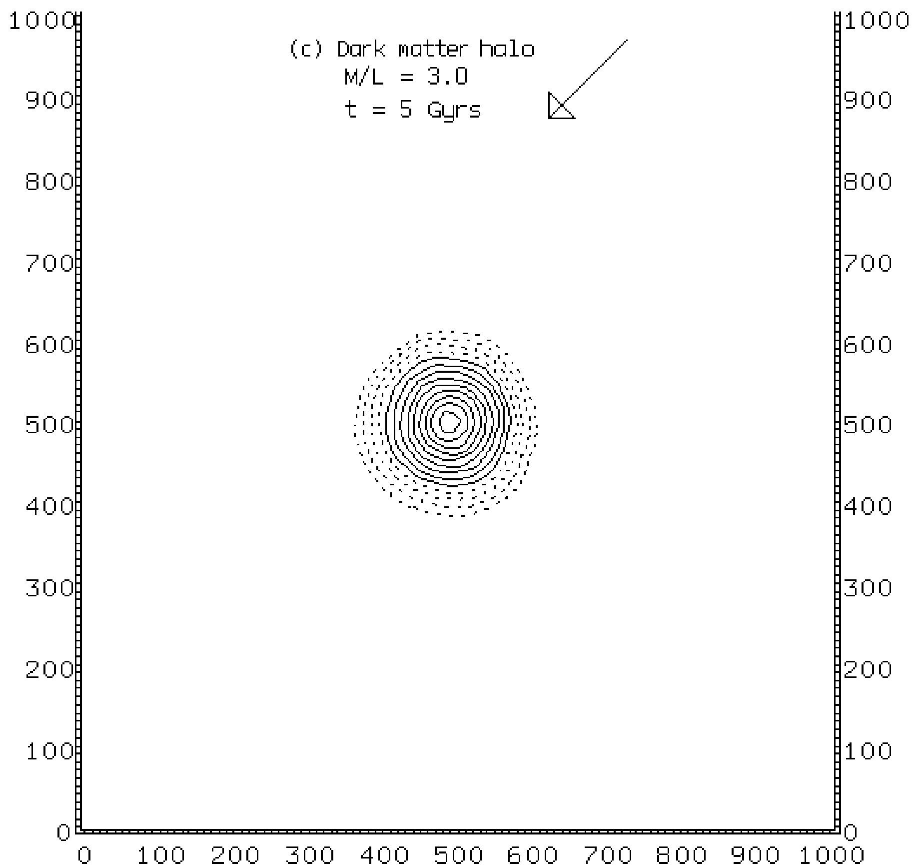
(c) Dark matter halo
M/L = 3.0
t = 5 Gyrs
0
100
200
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400
500
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700
800
900
1000

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Simulating a cluster for a Hubble time, even at our limited resolution requires more than 10^5 timesteps. However, since the rate of mass loss is constant we can compare our simulations with the observations of Grillmair *et al* after only a few Gyrs. Figure 1 shows a snapshot of a model with dark matter after 4 Gyrs. The tidal tails of dark matter are clearly visible since roughly 15% of the dark matter has reached the cluster escape velocity via relaxation of our artificially massive dark matter particles. Low mass stars or WIMPS will produce much smaller tidal tails since the relaxation timescale is proportional to $< m >^{-1/2}$.

The imaging data of Grillmair *et al* reveal of order a hundred stars above the background that are escaping from the cluster. This represents less than 1% of the total number of visible stars that are presently bound to the cluster. This is roughly the expected number of stars in the surveyed area for a typical cluster with no dark matter calculated using the evaporation rates from Johnston (1993) for a multi-component mass model.

In order to compare with the observations we smooth our data on a similar scale as Grillmair *et al* and plot iso-density contours of the stars. Figure 2 shows our results for the 3 models. Clearly, even modest amounts of dark matter can slow down or halt the stellar mass loss. The tidal tails from our standard model are quite similar to the observations, however, as the M/L ratios are increased the tails become much less prominent. Tidal extensions are still visible in our model which has a global M/L=2, although the stellar mass loss has been reduced by about a factor of 5 over the standard model with M/L=1. When we increased the mass to light ratio to 3, then the stellar mass loss is reduced to zero. From these simulations we conclude that the total mass to light ratios of globular clusters cannot be any higher than $\sim$ 2.5, otherwise escaping stars would not have been detected. (Similar results were obtained when we distributed the dark matter in an identical way to the stars.)

3.1 Numerical artifacts

How are these results effected by numerical resolution? We are using 10,000 mass points of effective size 1.5 pc to represent a star cluster that has an order of magnitude more stars of much smaller physical size. Evaporation is a diffusion process that is dominated by

distant encounters rather than close, strong scattering events. Therefore the fact that our stars are physically softened does not reduce the evaporation rate significantly. A more important artifact is a higher than expected mass loss rate since our star particle mass is effectively $20M_{\odot}$. This can only serve to underestimate our constraint on the global M/L values. Our treatment of the dark matter also enhances the evaporation rate of star particles since the dark particles are also artificially massive. As discussed earlier, in real clusters the dark matter is expected to be much lighter than the visible stars, thus we may be underestimating the effects of mass segregation. We do find that the half mass radius of the stars slowly decreases, although the rate of this effect is significantly smaller than analytic calculations yield (Spitzer 1987). More detailed simulations or analytic calculations could yield stronger constraints on the global M/L values.

§4. Discussion and conclusions

Symmetric tidal tails of stars being gravitationally stripped from globular clusters indicates that their mass distributions do not extend beyond the optical radius. This rules out the possibility of large dark matter halos surrounding globular clusters and hence excludes the possibility that these objects formed as density peaks within a cold dark matter type scenario (Peebles 1983, Rosenblatt 1988). We can also use these results to rule out the presence of large quantities of low mass stars either within or beyond the optical radii (Fahlman *et al* 1989, Richer *et al* 1991, Taillet *et al* 1995). A comparison of stellar isophotes beyond the tidal radii with the data of Grillmair *et al* (1995) provides an upper limit on the total mass to light ratios of globular clusters of M/L $\lesssim$3.

Tidal tails have also been observed to extend from the Ursa-Minor dwarf spheroidal (Irwin & Hatzidimitriou 1993). Data from these authors shows a large number of stars beyond the classical tidal radius of 400 pc. Our results can be readily applied to the dwarf spheroidal galaxies and suggests that the dark matter halo of Ursa-Minor must be truncated at the optical radius $\sim$ 400 pc and hence the mass distribution traces the light distribution.

If the dwarf spheroidal galaxies formed within cold dark matter halos then the fact that they are truncated at $\sim$ 400 pc has interesting consequences for cosmology. Namely, for the first time we have obtained an *upper* limit on the mass to light ratio of a dark matter dominated system. The radial velocity dispersion of stars within dwarf spheroidals appears to be fairly constant with radius If we parameterise the halo by an isothermal sphere with 1-d velocity dispersion 10 km s^{-1}, then the virial radius should nominally extend to 15 kpc for a closed Universe. Tidal truncation at the satellites present galacto-centric distance would limit the radial extent of its halo to $\sim$ 5 kpc. In order to strip the halo to within 400 pc would require a pericentric distance of $\sim$ 6 kpc. Proper motion studies of this galaxy would be extremely interesting and could be used to distinguish between halo formation

within a low density Universe, or tidal stripping on an elongated orbit.

Acknowledgments

I would like to acknowledge useful discussions with Fabio Governato and Doug Johnstone. The numerical simulations were performed and analysed using the resources of the HPCC group at Seattle, funded by NASA through the LTSA and HPCC/ESS programs.

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Figure captions

Figure 1. The particle distribution from one of our simulations after 4 Gyrs of evolution. The viewpoint is looking down upon the orbital plane and the curve depicts the cluster's orbit over 0.5 Gyrs. In this simulation the model globular cluster has a dark halo that extends beyond the stellar component. The presence of the dark halo has reduced the stellar mass loss to practically zero. The tidal tails contain 10% of the dark matter particles

which are lost primarily due to relaxation because of their artifically large masses. The inset plot shows the distribution of stars which shows no evidence of mass loss into tidal tails.

Figure 2. Contour plots of the stellar distribution in the immediate vicinity of the globular cluster which should be compared with Grillmair *et al*, in particular their Figure 13. The axis scale is in parsecs and the stellar density has been smoothed in projection using a Gaussian of width 5 pc. The arrows indicate the direction to the center of the galactic potential. (a) The standard model cluster without dark matter. (b) and (c) are for the model cluster with a dark matter halo such that the total mass to light ratios are 2 and 3 respectively. The similarity between the observations and the mass loss rate from our standard cluster in (a) provides conformation that our understanding of stellar dynamics and the evaporation process in star clusters is close to being correct.